SMP 16-19 Mathematics – Revision Notes
Unit 1 – Foundations

Graphs

1. Linear functions can be written in the form $y = mx + c$, whereby m is the gradient and c is the y-intercept.
2. Quadratic functions can be written in the form $y = ax^2 + bx + c$, and take the shape of a parabola.
3. The graph of $y = (x + p)^2 + q$ is a translation of the graph $y = x^2$ through the vector $\begin{bmatrix} -p \\ q \end{bmatrix}$.
4. To complete the square for the quadratic $ax^2 + bx + c$, write it in the form $(x + p)^2 + q$, whereby $p = \frac{1}{2}$, $q = -p^2$, then adjust the constant term by adding on c.
5. The roots of a quadratic are the solutions whereby $y = 0$, and can be seen on a graph by the x-intercepts.
6. To solve a quadratic algebraically, either manipulate the completed square form, factorise, or use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. To sketch a quadratic, the expanded form will give the y-intercept, the completed square form will give the vertex, and the factorised form will give the roots.

Sequences

1. An inductive definition of a sequence requires a starting value (i.e. u_1) and a recurrence relation (i.e. $u_{i+1} = f(u_i)$).
2. A sequence can converge to a fixed value either directly or it can be oscillatory.
3. A sequence that does not converge is said to diverge, and can diverge to infinity, can be periodic, or can be oscillatory.
4. It is often better to find a formula for the general term of a sequence, rather than rely on an inductive definition.
5. An arithmetic progression is a series whereby consecutive terms differ by a constant value.
6. The sum of an arithmetic progression can be calculated in two ways, whereby n is the number of terms, a is the first term, l is the last term, and d is the common difference:

$$S_n = \left(\frac{a+l}{2}\right)n$$
$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

7. The Annual Percentage Rate (APR) can be used to compare interest rates. It is the total interest on a year’s loan assuming the entire repayment is at the end of the year.
8. Sigma notation can be used to express a sequence in the form $\sum_{i=n}^{m} f(i)$.
9. A geometric progression is one where each term increases by a common ration (i.e. a multiple).
10. The sum of a geometric progression can be calculated as follows, whereby a is the first term, r is the common ration, and n is the number of terms:

$$\sum_{i=1}^{n} ar^{i-1} = a\frac{r^n - 1}{r - 1}$$

11. A geometric progression can have a sum to infinity when r is less than 1, and that sum can be calculated as follows:

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r} \quad for\ |r| < 1$$

Functions And Graphs

1. Function notation can be used to express a function of x in the form $f(x)$.
2. The graph of a function, $f(x + a) + b$ is the graph of $f(x)$ translated by the vector $\begin{bmatrix} -a \\ b \end{bmatrix}$.
3. The precise definition of a function consists of a rule, which tells you how to calculate values for the function, and a domain, which tells you the set of values for which you can apply the rule.
4. Important sets of numbers can be written in shorthand, whereby 'N' represents the set of natural numbers, 'Z' represents the set of integers, 'Q' represents the set of natural numbers and 'R' represents the set of real numbers. The notation '$^+$' and '$^-$' can also be used to represent the positive and the negative set respectively.
5. You can sketch a graph by having a good overall impression of its shape without the need to plot points in detail.
6. For a polynomial in the form $ax^m + \ldots + bx^n + c$, it will look like ax^m for very large values of x, and like $bx^n + c$ for very small values of x.
7. The ideas of dominance, the y-intercept, and the roots of the polynomial enable the general shape of a graph to be sketched easily.

Expressions And Equations

1. An identity is an algebraic statement that is true for any value of the variable, whereas an equation is only true for certain values, known as the solutions to the equation.
2. Quadratic equations can be solved by use of the equation $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
3. Inequalities can be manipulated using normal algebraic rules, except that when both sides are multiplied or divided by a negative number, the inequality sign is reversed.
4. Factor theorem states that if $x = a$ is a solution to the polynomial equation $P(x) = 0$, then $x - a$ is a factor of $P(x)$.
5. When factorising polynomials to the form $(x + n)(ax^2 + bx + c)$, then the value of a is given by the original value of a, the value of c is given by the original c divided by n, and the value of b can be given by equating the coefficients of x^2.

Numerical Methods

1. The methods of trial and improvement and iteration can both be used to solve equations numerically.
2. By sketching graphs using a graph plotter, it is possible to find the bounds of each of the solutions to an equation.
3. Iterative formulae can be used to calculate the value of a solution to an equation to a certain degree of accuracy.
4. Some iterative formulae may converge, whereas others may not. It is therefore important to test a variety of iterative formulae when attempting to numerically solve an equation.